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Regular-faced Convex Polyhedra

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ABSTRACT: *A regular-faced convex polyhedron is one in which all the faces are regular polygons, not necessarily with the same number of sides. Counting the prisms and antiprisms as one type of solid each, there seem to be 112 regular-faced convex polyhedra as conjectured by Johnson and later proved by Zalgaller. These polyhedra can all be built from combinations of 28 simple polyhedra plus prisms and antiprisms. The properties of these solids including the volume and sphericity are presented. The methods and problems in determining the number of these solids are given. Photographs of all the solids are shown. Nets for construction of the polyhedra are given, as well as methods of construction.*

I. Introduction

A classical mathematical problem since Euclid has been to classify polyhedra according to symmetry, number of vertices, connectivity, etc. (1). Classification schemes usually start with the most symmetrical polyhedra and work their way down to completely asymmetrical polyhedra with an increase in complexity as one decreases the symmetry requirements. The first type of polyhedron that is considered is usually one whose faces are identical regular polygons and whose vertices are all identical. The identity of vertices means that they can be transformed into each other by a symmetry operation (2). It is to be understood that in this article we restrict ourselves to three-dimensional Euclidean space. The above polyhedra are known as regular polyhedra and there are nine of them. Five of these are convex and are known as the Platonic solids. The four concave ones are known as the Kepler-Poinsot solids (3).

If one relaxes the restriction that all of the faces be identical but retains the requirement that they be regular polygons we obtain what Coxeter calls the uniform polyhedra (2). The convex, uniform polyhedra consist of the thirteen Archimedean solids and the classes of prisms and antiprisms. The number of concave uniform polyhedra is not known but Coxeter has determined that there are at least fifty-three and it is his opinion that there are no others (2). Grünbaum has listed several other types of "regular" polyhedra which can be obtained by relaxing restrictions on vertices or faces or

convexity (4). Fejes Tóth (5) and Coxeter (3) have listed and described the properties of many other "regular" polytopes in other than E^3 . In this paper we are concerned with only regular-faced convex polyhedra (hereafter denoted by *rfcp*). The two requirements for this type of polyhedra are that the solid be convex and that the faces be all regular polygons (not necessarily identical). Besides the Platonic and Archimedean solids and the two classes of prisms and antiprisms there are known to be 92 nonuniform *rfcp* (6). The method of arriving at the number of polyhedra, their properties and constructions will be discussed.

II. Number of Regular-faced Convex Polyhedra

Johnson was the first to produce a list of the *rfcp* (7). Johnson conjectured that the 112 polyhedra he listed (counting the prisms and the antiprisms as one polyhedron each) were the only possible ones. Zalgaller has written several papers on this subject (8, 9) and in 1967 gave an elaborate proof heavily based on combinatorial calculations performed on a computer, which indicates that Johnson's conjecture is correct (6).

In determining the number of *rfcp* we must utilize several well-known facts. In particular:

$$V - E + F = 2 \quad (\text{Euler's formula}), \quad (1)$$

$$0 < D_i = 360 - \sum A_{ij}, \quad (2)$$

$$\sum D_i = 720^\circ, \quad (3)$$

where V , E and F denote the number of vertices, edges and faces respectively in Eq. (1), A_{ij} denotes the face angle of the j th face at the i th vertex, and D_i denotes the deficiency of the i th vertex in Eqs. (2) and (3). Equations (1), (2) and (3) are necessary but not sufficient conditions for closure and convexity of polyhedra.

Employing the above facts, Grünbaum and Johnson (10) have proved two theorems that aid tremendously in enumerating the *rfcp*:

Theorem I. The number of nonuniform regular-faced convex polyhedra is finite.

Theorem II. The only polygons that may occur as faces of regular-faced convex polyhedra other than in prisms or antiprisms are triangles, squares, pentagons, hexagons, octagons and decagons.

With an application of Theorem II, it can be easily shown that the only possible vertices of a *rfcp* are those listed in Table I. Trivalent, quadravalent and pentavalent vertices are ones in which three, four or five faces, respectively, meet at a vertex. There obviously cannot be more than five faces meeting at a vertex. The notation used in Table I describes the possible cyclic arrangements of faces forming the vertices of a *rfcp*. If counter-clockwise and clockwise cyclic arrangements differ, then both the vertex and its mirror image are listed. The deficiency for each type of vertex is also given. The problem is to combine the vertices so that the sum of the deficiencies is

720°, Euler's formula is satisfied and the resulting solid is convex. We notice that the possible number of vertices can vary from 4 to 120.

Grace investigated the problem of determining the number of non-isomorphic trihedral convex polyhedra as a function of the number of faces. Grace performed a computer search by successive truncations of a tetrahedron. The method required several hours of computer time to search for trihedral convex polyhedra with up to 11 faces (11). The problem of enumerating the *rfcp* is not tractable by a method similar to Grace's since we must allow quadravalent and pentavalent, as well as combinations of these types of vertices. The search for *rfcp* is much more lengthy than Grace's and is obviously impractical on even the fastest computers available today. An alternative method to that offered by Grace will now be discussed.

TABLE I
Possible vertices of regular-faced convex polyhedra

Trivalent			Quadravalent			Pentavalent	
Vertex	Mirror	Def	Vertex	Mirror	Def	Vertex	Def
(3, 3, 3)		180	(3, 3, 3, 3)		120	(3, 3, 3, 3, 3)	60
(4, 3, 3)		150	(4, 3, 3, 3)		90	(4, 3, 3, 3, 3)	30
(4, 4, 3)		120	(4, 3, 4, 3)		60	(5, 3, 3, 3, 3)	12
(4, 4, 4)		90	(4, 4, 3, 3)		60		
(5, 3, 3)		132	(4, 4, 4, 3)		30		
(5, 4, 3)	(3, 4, 5)	102	(5, 3, 3, 3)		12		
(5, 4, 4)		72	(5, 3, 4, 3)		72		
(5, 5, 3)		84	(5, 3, 5, 3)		42		
(5, 5, 4)		54	(5, 4, 3, 3)	(3, 3, 4, 5)	24		
(5, 5, 5)		36	(5, 4, 3, 4)		42		
(6, 4, 3)	(3, 4, 6)	90	(5, 4, 4, 3)	(3, 4, 4, 5)	12		
(6, 4, 4)		60	(5, 5, 3, 3)		24		
(6, 5, 3)	(3, 5, 6)	72	(6, 3, 3, 3)		60		
(6, 5, 4)	(4, 5, 6)	42	(6, 3, 4, 3)		30		
(6, 5, 5)		24	(6, 3, 5, 3)		12		
(6, 6, 3)		60	(6, 4, 3, 3)	(3, 3, 4, 6)	30		
(6, 6, 4)		30	(6, 5, 3, 3)	(3, 3, 5, 6)	12		
(6, 6, 5)		12	(8, 3, 3, 3)		45		
(8, 4, 3)	(3, 4, 8)	75	(8, 3, 4, 3)		15		
(8, 4, 4)		45	(8, 4, 3, 3)	(3, 3, 4, 8)	15		
(8, 5, 3)	(3, 5, 8)	57	(10, 3, 3, 3)		36		
(8, 5, 4)	(4, 5, 8)	27	(10, 3, 4, 3)		6		
(8, 5, 5)		9	(10, 4, 3, 3)	(3, 3, 4, 10)	6		
(8, 6, 3)	(3, 6, 8)	45	(N, 3, 3, 3)		360/N		
(8, 6, 4)	(4, 6, 8)	15					
(8, 8, 3)		30					
(10, 4, 3)	(3, 4, 10)	66					
(10, 4, 4)		36					
(10, 5, 3)	(3, 5, 10)	48					
(10, 5, 4)	(4, 5, 10)	18					
(10, 6, 3)	(3, 6, 10)	36					
(10, 6, 4)	(4, 6, 10)	6					
(10, 8, 3)	(3, 8, 10)	21					
(10, 10, 3)		12					
(N, 4, 4)		360/N					

We recall that a trivalent vertex is rigid and the dihedral angles are determined by the face angles. If the trihedral vertex is denoted by (a, b, c) where a, b and c denote the number of sides of each face then the value of the dihedral angle A opposite face a is given by

$$\cos A = \frac{\cos [\pi(a-2)/a] - \cos [\pi(b-2)/b] \cos [\pi(c-2)/c]}{\sin [\pi(b-2)/b] \sin [\pi(c-2)/c]} \quad (4)$$

and other angles can be found by cyclic permutations of the above formula.

The dihedral angles for all of the possible trihedral vertices are given in Table II to the nearest microsecond. The vertices have to be combined into

TABLE II
Dihedral angles of possible trihedral vertices

Vertex	Dihedral angles					
(3, 3, 3)	70 31'	43-605716"	70 31'	43-605716"	70 31'	43-605716"
(4, 3, 3)	109 28'	16-394284"	54 44'	8-197142"	54 44'	8-197142"
(4, 4, 3)	90 0'	0-000000"	90 0'	0-000000"	60 0'	0-000000"
(4, 4, 4)	90 0'	0-000000"	90 0'	0-000000"	90 0'	0-000000"
(5, 3, 3)	138 11'	22-866375"	37 22'	38-525306"	37 22'	38-525306"
(5, 4, 3)	110 54'	18-566812"	79 11'	15-658931"	58 16'	57-092119"
(5, 4, 4)	108 0'	0-000000"	90 0'	0-000000"	90 0'	0-000000"
(5, 5, 3)	100 48'	44-341069"	100 48'	44-341069"	63 26'	5-815763"
(5, 5, 4)	108 57'	38-555575"	108 57'	38-555575"	96 3'	36-610312"
(5, 5, 5)	116 33'	54-184237"	116 33'	54-184237"	116 33'	54-184237"
(6, 4, 3)	125 15'	51-802858"	70 31'	43-605716"	54 44'	8-197142"
(6, 4, 4)	120 0'	0-000000"	90 0'	0-000000"	90 0'	0-000000"
(6, 5, 3)	114 48'	3-963108"	94 30'	47-635645"	65 11'	56-036892"
(6, 5, 4)	121 43'	2-907881"	110 54'	18-566812"	100 48'	44-341069"
(6, 5, 5)	131 10'	29-484549"	124 14'	53-136450"	124 14'	53-136450"
(6, 6, 3)	109 28'	16-394284"	109 28'	16-394284"	70 31'	43-605716"
(6, 6, 4)	125 15'	51-802858"	125 15'	51-802858"	109 28'	16-394284"
(6, 6, 5)	142 37'	21-474694"	142 37'	21-474694"	138 11'	22-866375"
(8, 4, 3)	144 44'	8-197142"	54 44'	8-197142"	45 0'	0-000000"
(8, 4, 4)	135 0'	0-000000"	90 0'	0-000000"	90 0'	0-000000"
(8, 5, 3)	132 8'	17-987330"	85 49'	45-593530"	65 15'	18-934097"
(8, 5, 4)	138 1'	48-305156"	115 54'	49-126271"	108 57'	38-555575"
(8, 5, 5)	152 32'	23-043223"	141 40'	3-283190"	141 40'	3-283190"
(8, 6, 3)	127 33'	5-770461"	103 50'	10-177725"	76 9'	49-822275"
(8, 6, 4)	144 44'	8-197142"	135 0'	0-000000"	125 15'	51-802858"
(8, 8, 3)	125 15'	51-802858"	125 15'	51-802858"	90 0'	0-000000"
(10, 4, 3)	159 5'	41-433188"	37 22'	38-525306"	31 43'	2-907881"
(10, 4, 4)	144 0'	0-000000"	90 0'	0-000000"	90 0'	0-000000"
(10, 5, 3)	142 37'	21-474694"	79 11'	15-658931"	63 26'	5-815763"
(10, 5, 4)	148 16'	57-092119"	121 43'	2-907881"	116 33'	54-184237"
(10, 6, 3)	138 11'	22-866375"	100 48'	44-341069"	79 11'	15-658931"
(10, 6, 4)	159 5'	41-433188"	148 16'	57-092119"	142 37'	21-474694"
(10, 8, 3)	138 3'	12-491245"	126 28'	24-997938"	99 59'	3-834591"
(10, 10, 3)	142 37'	21-474694"	142 37'	21-474694"	116 33'	54-184237"
(N, 4, 4)	180-360/N		90 0'	0-000000"	90 0'	0-000000"

a solid in such a way that they form a consistent set with compatible dihedral angles. For example, a $(3, 3, 3)$ vertex could not be connected to a $(3, 3, 5)$ vertex by a common edge because they have no compatible dihedral angles. In the case of solids made up solely of quadravalent or pentavalent vertices the situation is even more complicated because these vertices are not rigid and one has to determine the entire solid before one can calculate the dihedral angles, in order to determine if the solid is closed. For these three types of polyhedra we have:

trivalent *rfcp* satisfy

$$3F_3 + 2F_4 + F_5 = 12 + 2F_8 + 4F_{10}; \quad (5)$$

quadravalent *rfcp* satisfy

$$F_3 = 8 + F_5 + 2F_6 + 4F_8 + 6F_{10}; \quad (6)$$

and pentavalent *rfcp* satisfy

$$F_3 = 20 - 2F_4 + 5F_5; \quad (7)$$

where F_n is the number of n -gons. It should be noted that Euler's formula gives no information about the number of hexagons for trihedral solids (12) or squares for quadravalent solids. A simple enumeration on a computer of the number of combinations of F 's that satisfy Eqs. (5), (6) and (7) yields 23,801 trivalent, 1520 quadravalent and 49 pentavalent solutions. Since this count does not take into account the possible arrangement of the faces, the number of possible solids is obviously much greater. From the above number of solutions, determination of the number of combinations of faces for mixtures of types of vertices is seen to be intractable.

Zalgaller determined all the solutions of Eq. (5) that are consistent with Table II (6). These eleven solids are the tetrahedron, cube, dodecahedron, truncated tetrahedron, truncated octahedron, truncated cube, truncated icosahedron, truncated dodecahedron, truncated cuboctahedron, truncated icosidodecahedron and the prisms. All of these are uniform polyhedra. Zalgaller also showed that there are 29 *rfcp* which satisfy Eq. (6).

The 49 possible combinations of pentavalent vertices that satisfy Eq. (7) are consistent with the sum of the deficiencies being 720° and are presented in Table III. From Table I, the only possible pentavalent vertices are $(3, 3, 3, 3, M)$ where $M = 3, 4$ or 5 . Table III gives the number of triangles, squares, pentagons, faces, edges, vertices and vertices of type $(3, 3, 3, 3, M)$ for $M = 3, 4$ and 5 . The snub icosidodecahedron, snub cuboctahedron, snub square antiprism and the icosahedron are the only pentavalent *rfcp*. They correspond respectively to numbers 1, 7, 43 and 49 in Table III.

To determine other *rfcp* one must go through a tedious process, by trial and error, of considering all possible combinations of arrangements of faces. Then, for each possible arrangement of faces, one must perform elaborate computer computations to determine if the solid can be made closed or to prove closure impossible. Zalgaller performed the above calculations and claims that the *rfcp* have been found (6). We have verified some of his

TABLE III
Possible pentavalent regular-faced convex polyhedra

No.	n-gons			F	E	V	(3, 3, 3, 3, M)		
	3	4	5				3	4	5
1	80	0	12	92	150	60	0	0	60
2	72	1	10	83	135	54	0	4	50
3	64	2	8	74	120	48	0	8	40
4	56	3	6	65	105	42	0	12	30
5	48	4	4	56	90	36	0	16	20
6	40	5	2	47	75	30	0	20	10
7	32	6	0	38	60	24	0	24	0
8	75	0	11	86	140	56	1	0	55
9	67	1	9	77	125	50	1	4	45
10	59	2	7	68	110	44	1	8	35
11	51	3	5	59	95	38	1	12	25
12	43	4	3	50	80	32	1	16	15
13	35	5	1	41	65	26	1	20	5
14	70	0	10	80	130	52	2	0	50
15	62	1	8	71	115	46	2	4	40
16	54	2	6	62	100	40	2	8	30
17	46	3	4	53	85	34	2	12	20
18	38	4	2	44	70	28	2	16	10
19	30	5	0	35	55	22	2	20	0
20	65	0	9	74	120	48	3	0	45
21	57	1	7	65	105	42	3	4	35
22	49	2	5	56	90	36	3	8	25
23	41	3	3	47	75	30	3	12	15
24	33	4	1	38	60	24	3	16	5
25	60	0	8	68	110	44	4	0	40
26	52	1	6	59	95	38	4	4	30
27	44	2	4	50	80	32	4	8	20
28	36	3	2	41	65	26	4	12	10
29	28	4	0	32	50	20	4	16	0
30	55	0	7	62	100	40	5	0	35
31	47	1	5	53	85	34	5	4	25
32	39	2	3	44	70	28	5	8	15
33	31	3	1	35	55	22	5	12	5
34	50	0	6	56	90	36	6	0	30
35	42	1	4	47	75	30	6	4	20
36	34	2	2	38	60	24	6	8	10
37	26	3	0	29	45	18	6	12	0
38	45	0	5	50	80	32	7	0	25
39	37	1	3	41	65	26	7	4	15
40	29	2	1	32	50	20	7	8	5
41	40	0	4	44	70	28	8	0	20
42	32	1	2	35	55	22	8	4	10
43	24	2	0	26	40	16	8	8	0
44	35	0	3	38	60	24	9	0	15
45	27	1	1	29	45	18	9	4	5
46	30	0	2	32	50	20	10	0	10
47	22	1	0	23	35	14	10	4	0
48	25	0	1	26	40	16	11	0	5
49	20	0	0	20	30	12	12	0	0

calculations but verifying all of them by his method, much less by an independent method, is a gigantic task. We have constructed models of all the *rfcp* found by Johnson and have spent many frustrating hours trying to construct new solids. Invariably the resulting polyhedron is concave, has two adjacent faces parallel or, when calculations are performed, indicates that the apparent closure is due to bending or warping of the faces of the model. We are firmly convinced that Zalgaller is correct.

III. Names and Properties of the Regular-faced Convex Polyhedra

A summary of the properties of the *rfcp* is presented in Table IV. The area and volume for each polyhedron are given assuming an edge length of one. Photographs of the solids are given in Figs. 1–7 (the relative sizes of the polyhedra in the photographs are not comparable). Excellent photographs of models of the Platonic and Archimedean polyhedra as well as of several others have been published by Graziotti (13). The nets for construction of the solids are given in Figs. 8–26. In the case of the prisms and antiprisms only the simplest one is discussed for each (the extension of the properties to the general case is obvious). If one is interested in symmetry groups, dihedral angles and types of vertices for each polyhedron, the results are presented by Johnson (7). One interesting point regarding symmetry is that all of the solids possess a type of symmetry, i.e. none are completely assymmetric. This property can also be verified by examination of the nets. It is also of interest to note that only two-, three-, four- and fivefold axes of rotational symmetry occur in other than the prisms and antiprisms.

A simple solid is defined as one that cannot be dissected along one or more edges to produce two or more *rfcp*. Zalgaller has proved that all of the *rfcp* can be considered as made from combinations of 28 simple solids and prisms and antiprisms (6). The composition of all the solids is given in Table IV where the simple polyhedra are designated as $M1$ to $M28$ and the prisms and antiprisms by Pn and An , respectively. A bar over a solid indicates that the particular simple polyhedron is rotated. M 's and P 's are used in the figures.

The solids could be named in terms of their compositions of M 's, A 's and P 's. However, most of these polyhedra were named before Zalgaller proved that they could be made up of the simple solids, hence we will use the names adopted by Johnson (7). The seven solids that occur in two enantiomorphic (mirror image) forms are numbers 17, 18 and 64–68 in Table IV. The nets and photographs of only one of each of these is presented. The properties of the mirror image are easily deduced.

The names in Table IV can better be understood from the following definitions:

- (a) An n -gonal prism has n squares separating $2n$ -gons.
- (b) An n -gonal antiprism has $2n$ triangles separating $2n$ -gons.
- (c) An n -gonal pyramid has n triangles and one n -gonal base; n can be 3, 4 or 5.

TABLE IV
Properties of regular-faced convex polyhedra

No.	Name and composition	No. n -gons							F	E	V	Area	Volume	Sphere
		3	4	5	6	8	10							
1	Tetrahedron, $M1$	4	0	0	0	0	0	4	6	4	1.732051	0.117851	0.5498	
2	Octahedron, $2M2$	8	0	0	0	0	0	8	12	6	3.464102	0.471405	0.7776	
3	Cube, $P4$	0	6	0	0	0	0	6	12	8	6.000000	1.000000	0.7236	
4	Icosahedron, $A5+2M3 = M7+3M3$	20	0	0	0	0	0	20	30	12	8.660254	2.181695	0.9104	
5	Dodecahedron, $M15$	0	0	12	0	0	0	12	30	20	20.645729	7.663119	0.8687	
6	Cuboctahedron, $M4+\overline{M4}$	8	6	0	0	0	0	14	24	12	9.464102	2.357023	0.8609	
7	Icosidodecahedron, $M9+\overline{M9}$	20	0	12	0	0	0	32	60	30	29.305983	13.835526	0.9274	
8	Truncated tetrahedron, $M10$	4	0	0	4	0	0	8	18	12	12.124356	2.710576	0.6828	
9	Truncated octahedron, $M16$	0	6	0	8	0	0	14	36	24	26.784610	11.313708	0.8680	
10	Truncated cube, $M11$	8	0	0	0	6	0	14	36	24	32.434664	13.599663	0.7830	
11	Truncated icosahedron, $M19$	0	0	12	20	0	0	32	90	60	72.607253	55.287731	0.9504	
12	Truncated dodecahedron, $M12$	20	0	0	0	0	12	32	90	60	100.990760	85.039665	0.8911	
13	Rhombicuboctahedron, $M5+P8+M5$	8	18	0	0	0	0	26	48	24	21.464102	8.714045	0.9319	
14	Rhombicosidodecahedron, $M14+2M6 = M13+3M6$	20	30	12	0	0	0	62	120	60	59.305983	41.615324	0.9690	
15	Truncated cuboctahedron, $M17$	0	12	0	8	6	0	26	72	48	61.755172	41.798990	0.9160	
16	Truncated icosidodecahedron, $M18$	0	30	0	20	0	12	62	180	120	174.292030	206.803399	0.9558	
17	Snub cuboctahedron, $M26$	32	6	0	0	0	0	38	60	24	19.856406	7.889477	0.9483	
18	Snub icosidodecahedron, $M27$	80	0	12	0	0	0	92	150	60	55.286745	37.616650	0.9731	
19	Trigonal prism, $P3$	2	3	0	0	0	0	5	9	6	3.866025	0.433013	0.6058	
20	Quadrangular antiprism, $A4$	8	2	0	0	0	0	10	16	8	5.464102	0.957000	0.7968	
21	Square pyramid, $M2$	4	1	0	0	0	0	5	8	5	2.732051	0.235702	0.5551	
22	Pentagonal pyramid, $M3$	5	0	1	0	0	0	6	10	6	3.885541	0.301503	0.4186	
23	Triangular cupola, $M4$	4	3	0	1	0	0	8	15	9	7.330127	1.178511	0.6315	
24	Square cupola, $M5$	4	5	0	0	1	0	10	20	12	11.560478	1.942809	0.5256	
25	Pentagonal cupola, $M6$	5	5	1	0	0	1	12	25	15	16.579750	1.958072	0.3085	
26	Pentagonal rotunda, $M9$	10	0	6	0	0	1	17	35	20	22.347200	6.917763	0.6964	
27	Elongated triangular pyramid, $M1+P3$	4	3	0	0	0	0	7	12	7	4.732051	0.550864	0.5691	

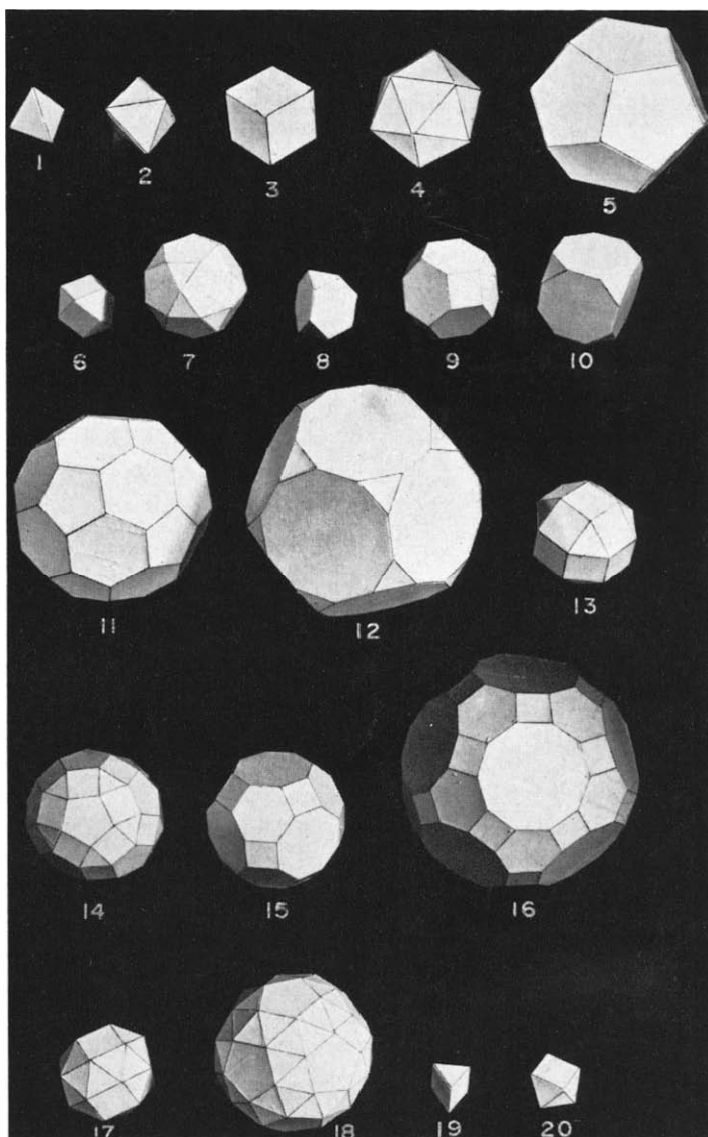


FIG. 1. Photographs of models of regular-faced convex polyhedra 1-20: 1, Tetrahedron M1; 2, Octahedron 2M2; 3, Cube P4; 4, Icosahedron $A5 + 2M3 = M7 + 3M3$; 5, Dodecahedron M15; 6, Cuboctahedron $M4 + \overline{M4}$; 7, Icosidodecahedron $M9 + \overline{M9}$; 8, Truncated tetrahedron M10; 9, Truncated octahedron M16; 10, Truncated cube M11; 11, Truncated icosahedron M19; 12, Truncated dodecahedron M12; 13, Rhombicuboctahedron $M5 + P8 + M5$; 14, Rhombicosidodecahedron $M14 + 2M6 = M13 + 3M6$; 15, Truncated cuboctahedron M17; 16, Truncated icosidodecahedron M18; 17, Snub cuboctahedron M26; 18, Snub icosidodecahedron M27; 19, Trigonal prism P3; 20, Quadrangular antiprism A4.

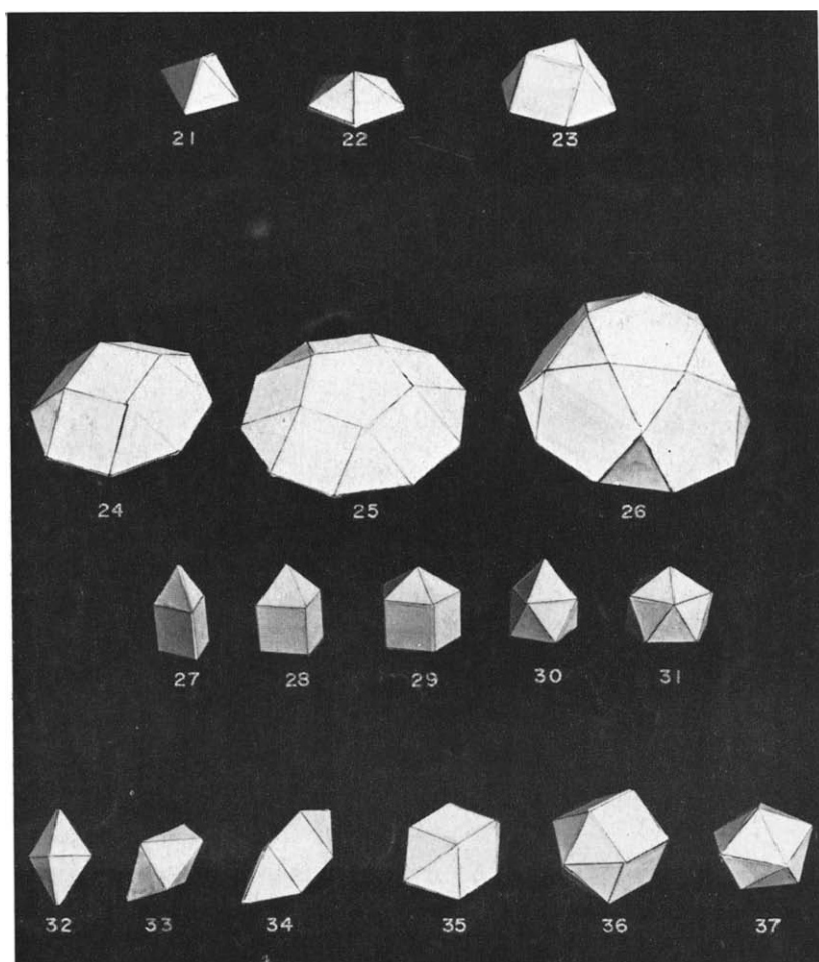


FIG. 2. Photographs of models of regular-faced convex polyhedra 21-37: 21, Square pyramid M2; 22, Pentagonal pyramid M3; 23, Triangular cupola M4; 24, Square cupola M5; 25, Pentagonal cupola M6; 26, Pentagonal rotunda M9; 27, Elongated triangular pyramid M1+P3; 28, Elongated square pyramid M2+P4; 29, Elongated pentagonal pyramid M3+P5; 30, Gyroelongated square pyramid M2+A4; 31, Gyroelongated pentagonal pyramid M3+A5; 32, Triangular dipyrmaid 2M1; 33, Pentagonal dipyrmaid 2M3; 34, Elongated triangular dipyrmaid M1+P3+M1; 35, Elongated square dipyrmaid M2+P4+M2; 36, Elongated pentagonal dipyrmaid M3+P5+M3; 37, Gyroelongated square dipyrmaid M2+A4+M2.

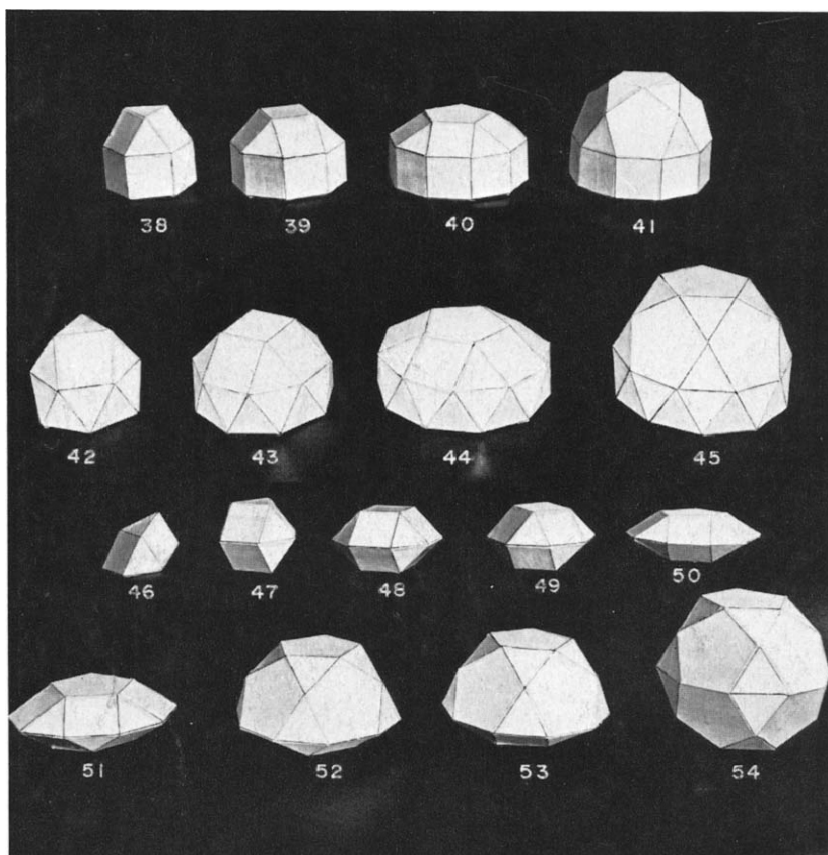


FIG. 3. Photographs of models of regular-faced convex polyhedra 38–54: 38, Elongated triangular cupola $M_4 + P_6$; 39, Elongated square cupola $M_5 + P_8$; 40, Elongated pentagonal cupola $M_6 + P_{10}$; 41, Elongated pentagonal rotunda $M_9 + P_{10}$; 42, Gyroelongated triangular cupola $M_5 + A_6$; 43, Gyroelongated square cupola $M_5 + A_8$; 44, Gyroelongated pentagonal cupola $M_6 + A_{10}$; 45, Gyroelongated pentagonal rotunda $M_9 + A_{10}$; 46, Gyrobifastigium $P_3 + \bar{P}_3$; 47, Triangular orthobicupola $2M_4$; 48, Square orthobicupola $2M_5$; 49, Square gyrobicupola $M_5 + \bar{M}_5$; 50, Pentagonal orthobicupola $2M_6$; 51, Pentagonal gyrobicupola $M_6 + \bar{M}_6$; 52, Pentagonal orthocupolarotunda $M_6 + \bar{M}_9$; 53, Pentagonal gyrocupolarotunda $M_6 + \bar{M}_9$; 54, Pentagonal orthobirotunda $2M_9$.

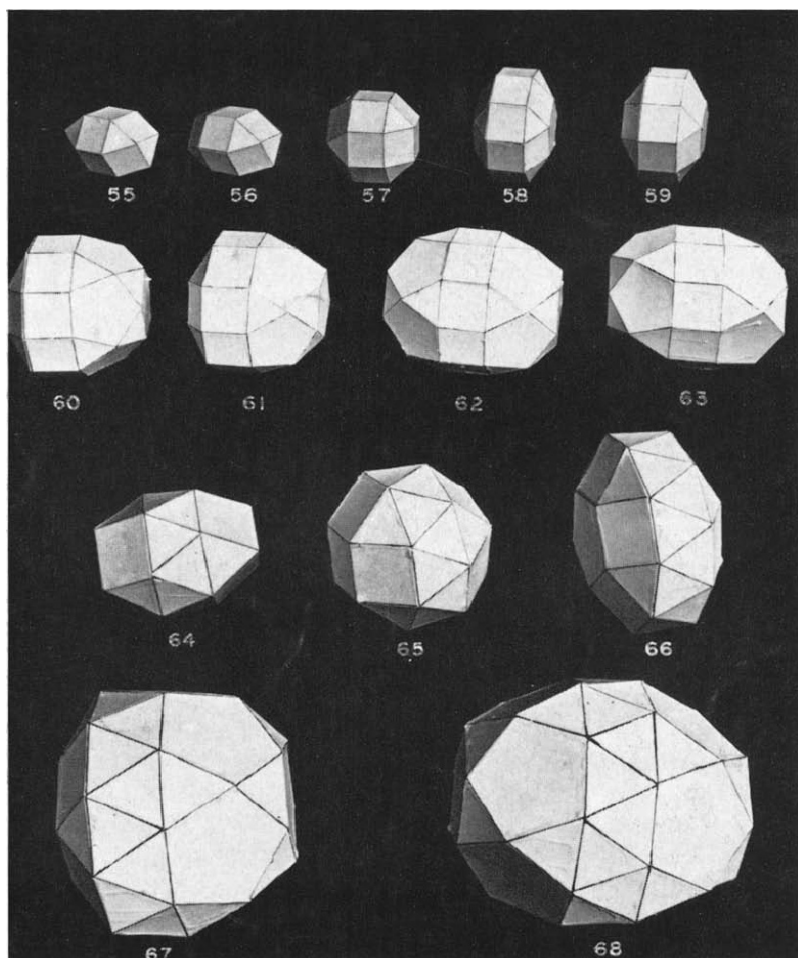


FIG. 4. Photographs of models of regular-faced convex polyhedra 55–68: 55, Elongated triangular orthobicupola $M_4 + P_6 + M_4$; 56, Elongated triangular gyrobicupola $M_4 + P_6 + \overline{M}_4$; 57, Elongated square gyrobicupola $M_5 + P_8 + \overline{M}_5$; 58, Elongated pentagonal orthobicupola $M_6 + P_{10} + M_6$; 59, Elongated pentagonal gyrobicupola $M_6 + P_{10} + \overline{M}_6$; 60, Elongated pentagonal orthocupolarotunda $M_6 + P_{10} + M_9$; 61, Elongated pentagonal gyrocupolarotunda $M_6 + P_{10} + \overline{M}_9$; 62, Elongated pentagonal orthobirotunda $M_9 + P_{10} + M_9$; 63, Elongated pentagonal gyrobirotunda $M_9 + P_{10} + \overline{M}_9$; 64, Gyroelongated triangular bicupola $M_4 + A_6 + M_4$; 65, Gyroelongated square bicupola $M_5 + A_8 + M_5$; 66, Gyroelongated Pentagonal Bicupola $M_6 + A_{10} + M_6$; 67, Gyroelongated pentagonal cupolarotunda $M_6 + A_{10} + M_9$; 68, Gyroelongated pentagonal birotunda $M_9 + A_{10} + M_9$.

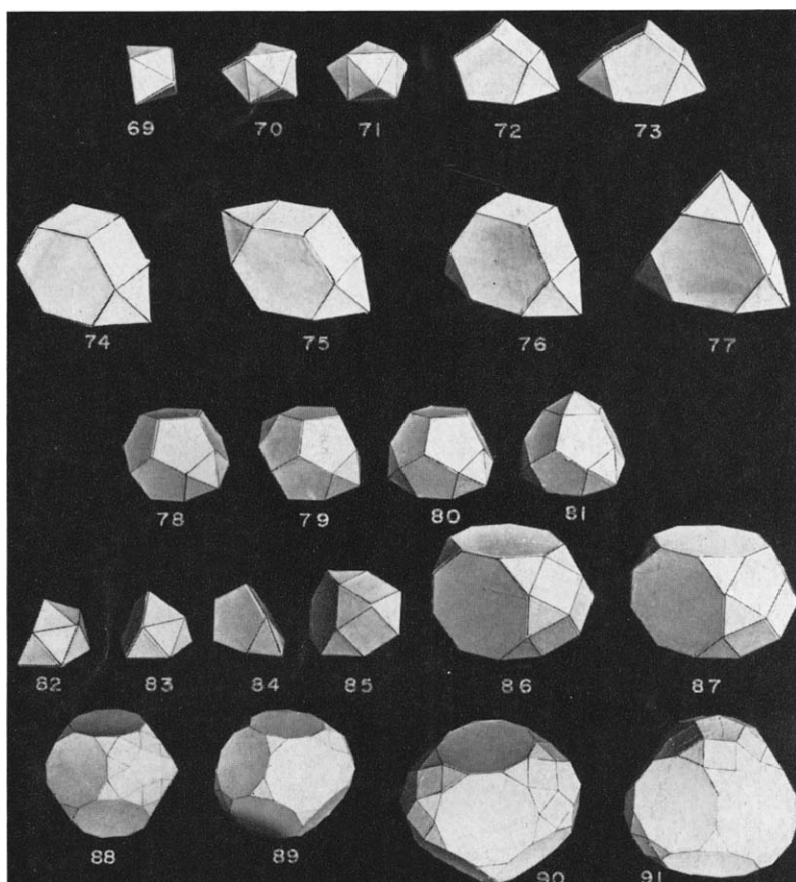


FIG. 5. Photographs of models of regular-faced convex polyhedra 69–91: 69, Augmented triangular prism $P3+M2$; 70, Biaugmented triangular prism $P3+2M2$; 71, Triaugmented triangular prism $P3+3M2$; 72, Augmented pentagonal prism $P5+M2$; 73, Biaugmented pentagonal prism $P5+2M2$; 74, Augmented hexagonal prism $P6+M2$; 75, Parabiaugmented hexagonal prism $M2+P6+M2$; 76, Metabiaugmented hexagonal prism $P6+2M2$; 77, Triaugmented hexagonal prism $P6+3M2$; 78, Augmented dodecahedron $M15+M3$; 79, Parabiaugmented dodecahedron $M3+M15+M3$; 80, Metabiaugmented dodecahedron $M15+2M3$; 81, Triaugmented dodecahedron $M15+3M3$; 82, Metabidiminished icosahedron $M7+M3$; 83, Tridiminished icosahedron $M7$; 84, Augmented tridiminished icosahedron $M7+M1$; 85, Augmented truncated tetrahedron $M10+M4$; 86, Augmented truncated cube $M11+M5$; 87, Biaugmented truncated cube $M5+M11+M5$; 88, Augmented truncated dodecahedron $M6+M12$; 89, Parabiaugmented truncated dodecahedron $M6+M12+M6$; 90, Metabiaugmented truncated dodecahedron $M12+2M6$; 91, Triaugmented truncated dodecahedron $M12+3M6$.

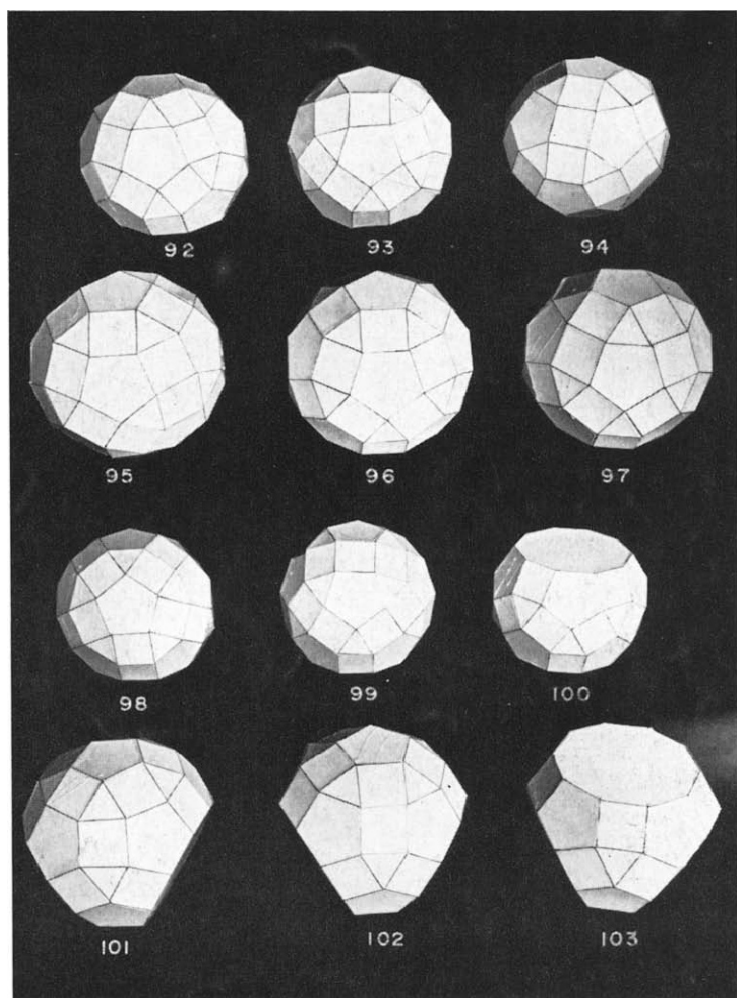


FIG. 6. Photographs of models of regular-faced convex polyhedra 92-103: 92, Gyrate rhombicosidodecahedron $\overline{M6} + M14 + M6 = \overline{M6} + M13 + 2M6$; 93, Parabigyrate rhombicosidodecahedron $\overline{M6} + M14 + \overline{M6}$; 94, Metabigyrate rhombicosidodecahedron $2\overline{M6} + M13 + M6$; 95, Trigyrate rhombicosidodecahedron $3\overline{M6} + M13$; 96, Diminished rhombicosidodecahedron $M6 + M14 = 2M6 + M13$; 97, Paragyrate diminished rhombicosidodecahedron $M14 + \overline{M6}$; 98, Metagyrate diminished rhombicosidodecahedron $M13 + M6 + \overline{M6}$; 99, Bigyrate diminished rhombicosidodecahedron $M13 + 2\overline{M6}$; 100, Parabidiminished rhombicosidodecahedron $M14$; 101, Metabidiminished rhombicosidodecahedron $M13 + M6$; 102, Gyrate bidiminished rhombicosidodecahedron $M13 + \overline{M6}$; 103, Tridiminished rhombicosidodecahedron $M13$.

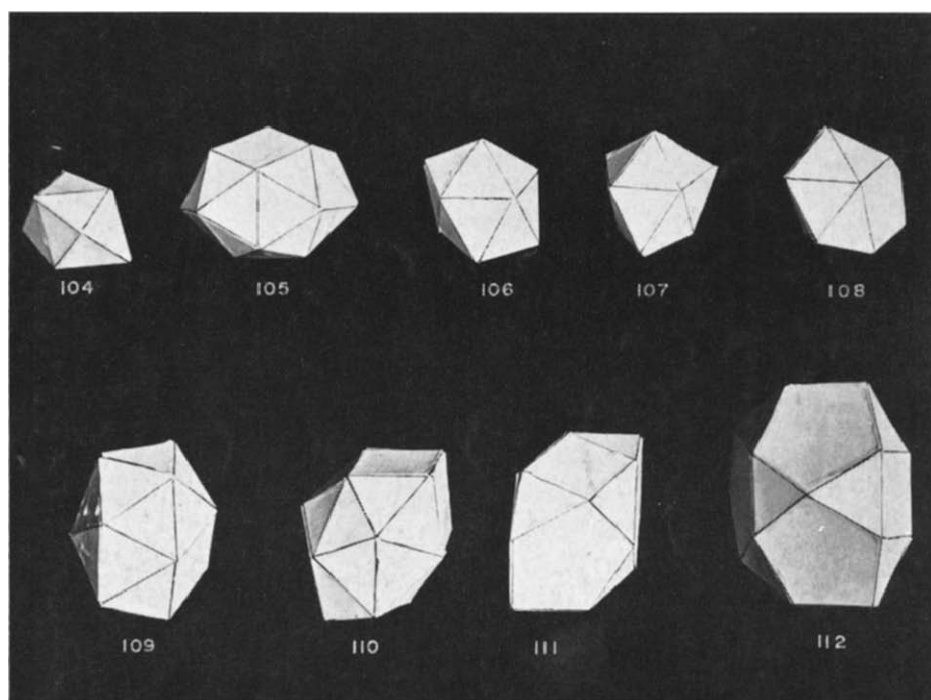


FIG. 7. Photographs of models of regular-faced convex polyhedra 104–112: 104, Snub disphenoid M25; 105, Snub square antiprism M28; 106, Sphenocorona M22; 107, Augmented sphenocorona $M22 + M2$; 108, Sphenomegacorona M23; 109, Hebesphenomegacorona M21; 110, Disphenocingulum M24; 111, Bilunabirotonda M8; 112, Triangular hebesphenorotunda M20.

28	Elongated square pyramid, $M2 + P4$	4	5	0	0	0	0	0	0	9	16	9	6-732051	1-235702	0-7523
29	Elongated pentagonal pyramid, $M3 + P5$	5	5	1	0	0	0	0	0	11	20	11	8-885541	2-021980	0-8119
30	Gyroelongated square pyramid, $M2 + A4$	12	1	0	0	0	0	0	0	13	20	9	6-196152	1-192702	0-8224
31	Gyroelongated pentagonal pyramid, $M3 + A5$	15	0	1	0	0	0	0	0	16	25	11	8-215668	1-880192	0-8491
32	Triangular dipyrmaid, $2M1$	6	0	0	0	0	0	0	0	6	9	5	2-598076	0-235702	0-5986
33	Pentagonal dipyrmaid, $2M3$	10	0	0	0	0	0	0	0	10	15	7	4-330127	0-603006	0-7117
34	Elongated triangular dipyrmaid, $M1 + P3 + M1$	6	3	0	0	0	0	0	0	9	15	8	5-598076	0-668715	0-5369
35	Elongated square dipyrmaid, $M2 + P4 + M2$	8	4	0	0	0	0	0	0	12	20	10	7-464102	1-471405	0-7673
36	Elongated pentagonal dipyrmaid, $M3 + P5 + M3$	10	5	0	0	0	0	0	0	15	25	12	9-330127	2-323483	0-8670
37	Gyroelongated square dipyrmaid, $M2 + A4 + M2$	16	0	0	0	0	0	0	0	16	24	10	6-928203	1-428405	0-8330
38	Elongated triangular cupola, $M4 + P6$	4	9	0	1	0	0	0	0	14	27	15	13-330127	3-776588	0-8252
39	Elongated square cupola, $M5 + P8$	4	13	0	0	1	0	0	0	18	36	20	19-560478	6-771236	0-8324
40	Elongated pentagonal cupola, $M6 + P10$	5	15	1	0	0	0	0	0	22	45	25	26-579750	9-652281	0-7491
41	Elongated pentagonal rotunda, $M9 + P10$	10	10	6	0	0	0	0	1	27	55	30	32-347200	14-611972	0-8447
42	Gyroelongated triangular cupola, $M4 + A6$	16	3	0	1	0	0	0	0	20	33	15	12-526279	3-516053	0-8434
43	Gyroelongated square cupola, $M5 + A8$	20	5	0	0	1	0	0	0	26	44	20	18-488681	6-210766	0-8308
44	Gyroelongated pentagonal cupola, $M6 + A10$	25	5	1	0	0	0	1	0	32	55	25	25-240004	8-707360	0-7303
45	Gyroelongated pentagonal rotunda, $M9 + A10$	30	0	6	0	0	0	0	1	37	65	30	31-007454	13-667051	0-8418
46	Gyrobifastigium, $P3 + \overline{P3}$	4	4	0	0	0	0	0	0	8	14	8	5-732051	0-866025	0-6711
47	Triangular orthobicupola, $2M4$	8	6	0	0	0	0	0	0	14	24	12	9-464102	2-357023	0-8609
48	Square orthobicupola, $2M5$	8	10	0	0	0	0	0	0	18	32	16	13-464102	3-885618	0-8364
49	Square gyrobicupola, $M5 + \overline{M5}$	8	10	0	0	0	0	0	0	18	32	16	13-464102	3-885618	0-8364
50	Pentagonal orthobicupola, $2M6$	10	10	2	0	0	0	0	0	22	40	20	17-771082	3-916144	0-5559
51	Pentagonal gyrobicupola, $M6 + \overline{M6}$	10	10	2	0	0	0	0	0	22	40	20	17-771082	3-916144	0-5559
52	Pentagonal orthocubolatorunda, $M6 + M9$	15	5	7	0	0	0	0	0	27	50	25	23-538532	8-875835	0-8265
53	Pentagonal gyrocubolatorunda, $M6 + \overline{M9}$	15	5	7	0	0	0	0	0	27	50	25	23-538532	8-875835	0-8265
54	Pentagonal orthobirotunda, $2M9$	20	0	12	0	0	0	0	0	32	60	30	29-305983	13-835526	0-9274
55	Elongated triangular orthobicupola, $M4 + P6 + M4$	8	12	0	0	0	0	0	0	20	36	18	15-464102	4-955099	0-8665

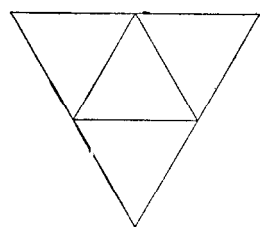
TABLE IV (cont.)
Properties of regular-faced convex polyhedra

No.	Name and composition	No. n-gons							F	E	V	Area	Volume	Sphere
		3	4	5	6	8	10							
56	Elongated triangular gyrobicupola, $M4 + P6 + \overline{M}4$	8	12	0	0	0	0	20	36	18	15.464102	4.955099	0.8665	
57	Elongated square gyrobicupola, $M5 + P8 + \overline{M}5$	8	18	0	0	0	0	26	48	24	21.464102	8.714045	0.9319	
58	Elongated pentagonal orthobicupola, $M6 + P10 + M6$	10	20	2	0	0	0	32	60	30	27.771082	11.610353	0.8437	
59	Elongated pentagonal gyrobicupola $M6 + P10 + \overline{M}6$	10	20	2	0	0	0	32	60	30	27.771082	11.610353	0.8437	
60	Elongated pentagonal orthocupolarotunda, $M6 + P10 + M9$	15	15	7	0	0	0	37	70	35	33.538532	16.570044	0.9073	
61	Elongated pentagonal gyrocupolarotunda, $M6 + P10 + \overline{M}9$	15	15	7	0	0	0	37	70	35	33.538532	16.570044	0.9073	
62	Elongated pentagonal orthobirotunda, $M9 + P10 + M9$	20	10	12	0	0	0	42	80	40	39.305983	21.529735	0.9291	
63	Elongated pentagonal gyrobirotunda, $M9 + P10 + \overline{M}9$	20	10	12	0	0	0	42	80	40	39.305983	21.529735	0.9291	
64	Gyroelongated triangular bicupola, $M4 + A6 + M4$	20	6	0	0	0	0	26	42	18	14.660254	4.694564	0.8894	
65	Gyroelongated square bicupola, $M5 + A8 + M5$	24	10	0	0	0	0	34	56	24	20.392305	8.153575	0.9416	
66	Gyroelongated pentagonal bicupola, $M6 + A10 + M6$	30	10	2	0	0	0	42	70	30	26.431336	10.665432	0.8347	
67	Gyroelongated pentagonal cupolarotunda, $M6 + A10 + M9$	35	5	7	0	0	0	47	80	35	32.198786	15.625123	0.9095	
68	Gyroelongated pentagonal birotunda, $M9 + A10 + M9$	40	0	12	0	0	0	52	90	40	37.966237	20.584814	0.9358	
69	Augmented triangular prism, $P3 + M2$	6	2	0	0	0	0	8	13	7	4.598076	0.668715	0.7213	
70	Biaugmented triangular prism, $P3 + 2M2$	10	1	0	0	0	0	11	17	8	5.330127	0.904417	0.7816	
71	Triaugmented triangular prism $P3 + 3M2$	14	0	0	0	0	0	14	21	9	6.062178	1.140119	0.8123	

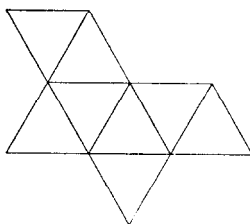
72	Augmented pentagonal prism, $P5 + M2$	4	4	2	0	0	0	10	19	11	9-173006	1-956180	0-7488
73	Biaugmented pentagonal prism, $P5 + 2M2$	8	3	2	0	0	0	13	23	12	9-905056	2-191882	0-7478
74	Augmented hexagonal prism, $P6 + M2$	4	5	0	2	0	0	11	22	13	11-928203	2-833778	0-7315
75	Parabiaugmented hexagonal prism, $M2 + P6 + M2$	8	4	0	2	0	0	14	26	14	12-660254	3-069481	0-7246
76	Metabiaugmented hexagonal prism, $P6 + 2M2$	8	4	0	2	0	0	14	26	14	12-660254	3-069481	0-7246
77	Triaugmented hexagonal prism, $P6 + 3M2$	12	3	0	2	0	0	17	30	15	13-392305	3-305183	0-7172
78	Augmented dodecahedron, $M15 + M3$	5	0	11	0	0	0	16	35	21	21-090315	7-964622	0-8745
79	Parabiaugmented dodecahedron, $M3 + M15 + M3$	10	0	10	0	0	0	20	40	22	21-534901	8-266125	0-8797
80	Metabiaugmented dodecahedron, $M15 + 2M3$	10	0	10	0	0	0	20	40	22	21-534901	8-266125	0-8797
81	Triaugmented dodecahedron, $M15 + 3M3$	15	0	9	0	0	0	24	45	23	21-979487	8-567627	0-8842
82	Metabidiminished icosahedron, $M7 + M3$	10	0	2	0	0	0	12	20	10	7-771082	1-578689	0-7750
83	Tridiminished icosahedron, $M7$	5	0	3	0	0	0	8	15	9	7-326496	1-277186	0-6849
84	Augmented tridiminished icosahedron, $M7 + M1$	7	0	3	0	0	0	10	18	10	8-192521	1-395038	0-6327
85	Augmented truncated tetrahedron, $M10 + M4$	8	3	0	3	0	0	14	27	15	14-253330	3-889087	0-7682
86	Augmented truncated cube, $M11 + M5$	12	5	0	0	5	0	22	48	28	34-338288	15-542472	0-8214
87	Biaugmented truncated cube, $M5 + M11 + M5$	16	10	0	0	4	0	30	60	32	36-241912	17-485281	0-8523
88	Augmented truncated dodecahedron, $M6 + M12$	25	5	1	0	0	11	42	105	65	102-182092	86-997737	0-8957
89	Parabiaugmented truncated dodecahedron, $M6 + M12 + M6$	30	10	2	0	0	10	52	120	70	103-373424	88-955809	0-9001
90	Metabiaugmented truncated dodecahedron, $M12 + 2M6$	30	10	2	0	0	10	52	120	70	103-373424	88-955809	0-9001
91	Triaugmented truncated dodecahedron, $M12 + 3M6$	35	15	3	0	0	9	62	135	75	104-564756	90-913881	0-9042
92	Gyrate rhombicosidodecahedron, $M6 + M14 + M6 = M6 + M13 + 2M6$	20	30	12	0	0	0	62	120	60	59-305983	41-615324	0-9690
93	Parabigyrate rhombicosidodecahedron, $M6 + M14 + M6$	20	30	12	0	0	0	62	120	60	59-305983	41-615324	0-9690

TABLE IV (cont.)
Properties of regular-faced convex polyhedra

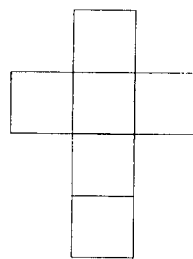
No.	Name and composition	No. n -gons							F	E	V	Area	Volume	Sphere
		3	4	5	6	8	10							
94	Metabigyrate rhombicosidodecahedron, $2M6 + M13 + M\overline{6}$	20	30	12	0	0	0	62	120	60	59.305983	41.615324	0.9690	
95	Trigyrate rhombicosidodecahedron, $3M\overline{6} + M13$	20	30	12	0	0	0	62	120	60	59.305983	41.615324	0.9690	
96	Diminished rhobicosidodecahedron, $M6 + M14 = 2M6 + M13$	15	25	11	0	0	1	52	105	55	58.114651	39.657252	0.9520	
97	Paragyrate diminished rhombicosidodecahedron, $M14 + M\overline{6}$	15	25	11	0	0	1	52	105	55	58.114651	39.657252	0.9520	
98	Metagyrate diminished rhombicosidodecahedron, $M13 + M6 + M\overline{6}$	15	25	11	0	0	1	52	105	55	58.114651	39.657252	0.9520	
99	Bigyrate diminished rhombicosidodecahedron, $M13 + 2M\overline{6}$	15	25	11	0	0	1	52	105	55	58.114651	39.657252	0.9520	
100	Parabidiminished rhombicosidodecahedron, $M14$	10	20	10	0	0	2	42	90	50	56.923319	37.699179	0.9335	
101	Metabidiminished rhombicosidodecahedron, $M13 + M6$	10	20	10	0	0	2	42	90	50	56.923319	37.699179	0.9335	
102	Gyrate bidiminished rhombicosidodecahedron $M13 + M\overline{6}$	10	20	10	0	0	2	42	90	50	56.923319	37.699179	0.9335	
103	Tridiminished rhombicosidodecahedron, $M13$	5	15	9	0	0	3	32	75	45	55.731987	35.741107	0.9136	
104	Snub disphenoid, $M25$	12	0	0	0	0	0	12	18	8	5.196152	0.859494	0.7717	
105	Snub square antiprism, $M28$	24	2	0	0	0	0	26	40	16	12.392305	3.601222	0.8779	
106	Sphenocorona, $M22$	12	2	0	0	0	0	14	22	10	7.196152	1.515332	0.8348	
107	Augmented sphenocorona, $M22 + M2$	16	1	0	0	0	0	17	26	11	7.928203	1.751034	0.8342	
108	Sphenomegacorona, $M23$	16	2	0	0	0	0	18	28	12	8.928203	1.948108	0.7766	
109	Hebesphenomegacorona, $M21$	18	3	0	0	0	0	21	33	14	10.794229	2.912912	0.8735	
110	Disphenocingulum, $M24$	20	4	0	0	0	0	24	38	16	12.660254	3.777645	0.8918	
111	Bilunabirotunda, $M8$	8	2	4	0	0	0	14	26	14	12.346011	3.093718	0.7584	
112	Triangular hebesphenorotunda, $M20$	13	3	3	1	0	0	20	36	18	16.388674	5.108746	0.8189	



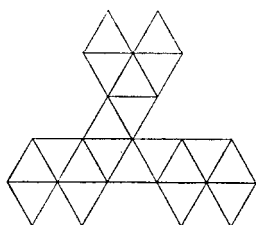
1. Tetrahedron M1



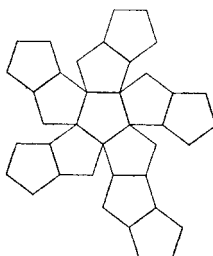
2. Octahedron 2M2



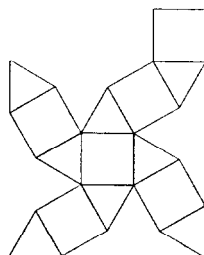
3. Cube P4



4. Icosahedron $A5 + 2M3 + M7 + 3M3$

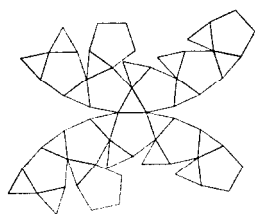


5. Dodecahedron M15

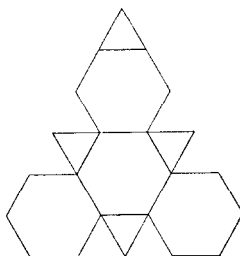


6. Cuboctahedron $M4 + \overline{M4}$

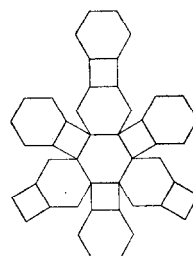
FIG. 8. Nets of regular-faced convex polyhedra 1-6.



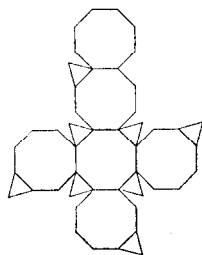
7. Icosidodecahedron $M9 + \overline{M9}$



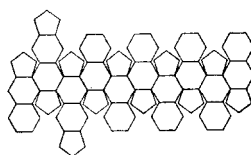
8. Truncated tetrahedron M10



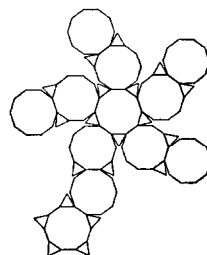
9. Truncated octahedron M16



10. Truncated cube M11



11. Truncated icosahedron M19



12. Truncated dodecahedron M12

FIG. 9. Nets of regular-faced convex polyhedra 7-12.

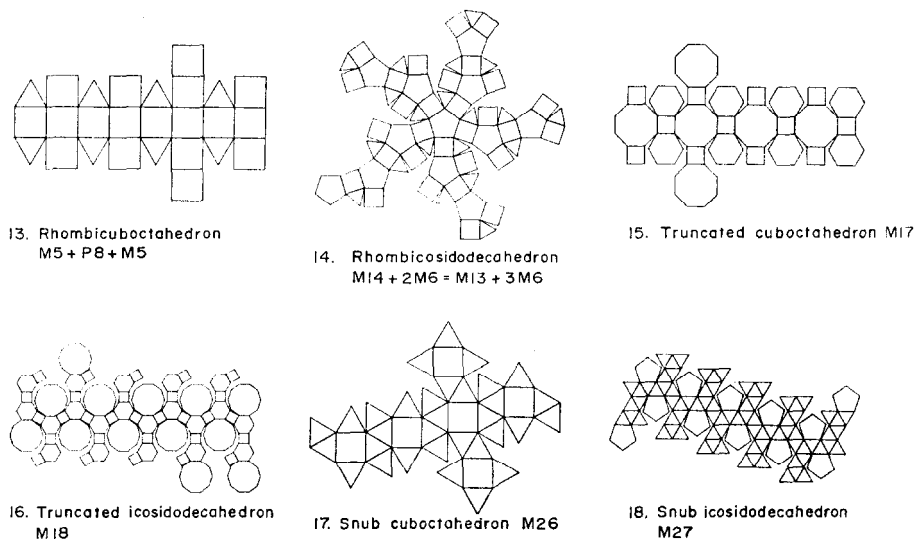


FIG. 10. Nets of regular-faced convex polyhedra 13–18.

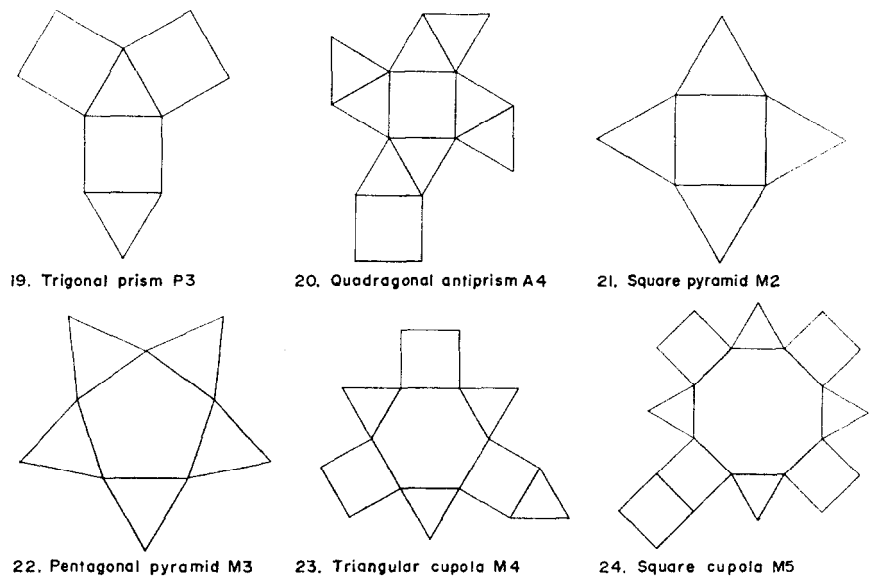


FIG. 11. Nets of regular-faced convex polyhedra 19–24.

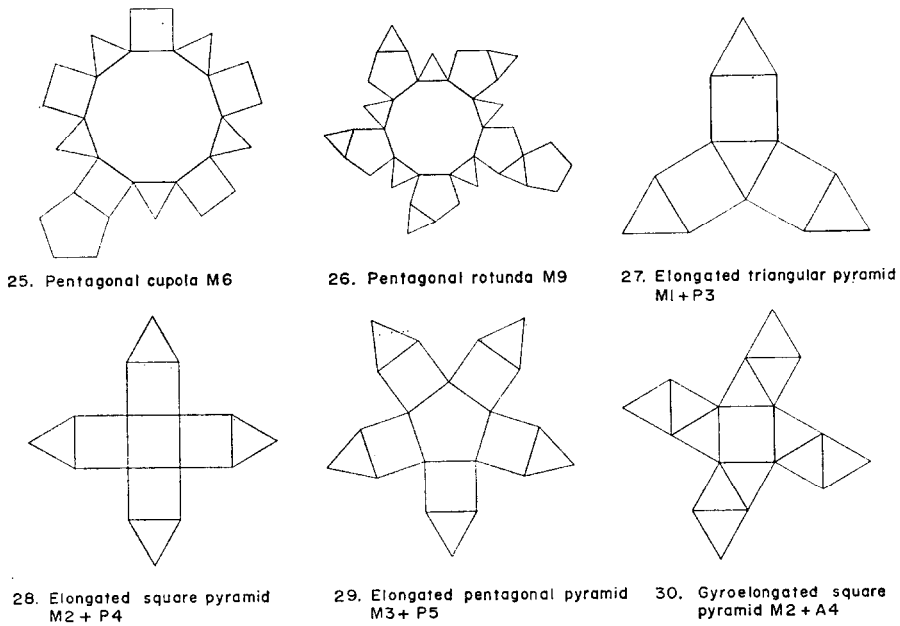


FIG. 12. Nets of regular-faced convex polyhedra 25-30.

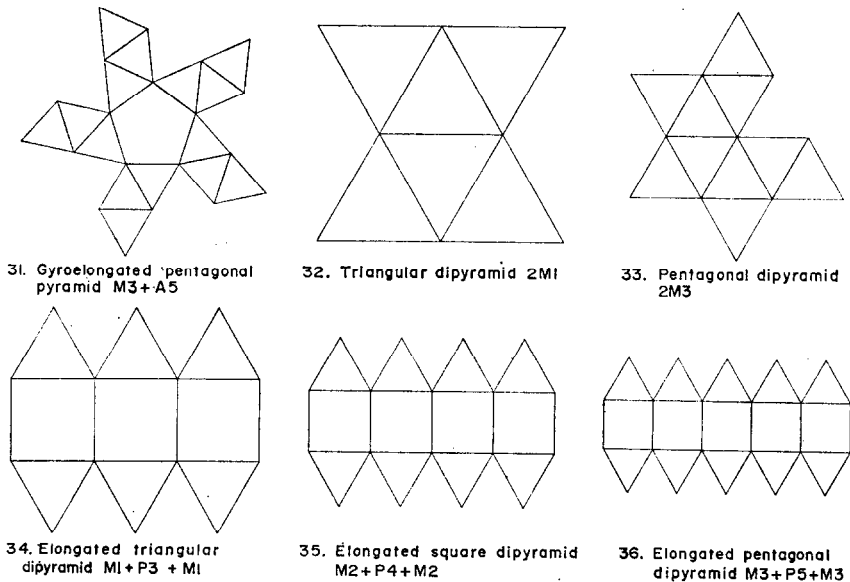


FIG. 13. Nets of regular-faced convex polyhedra 31-36.

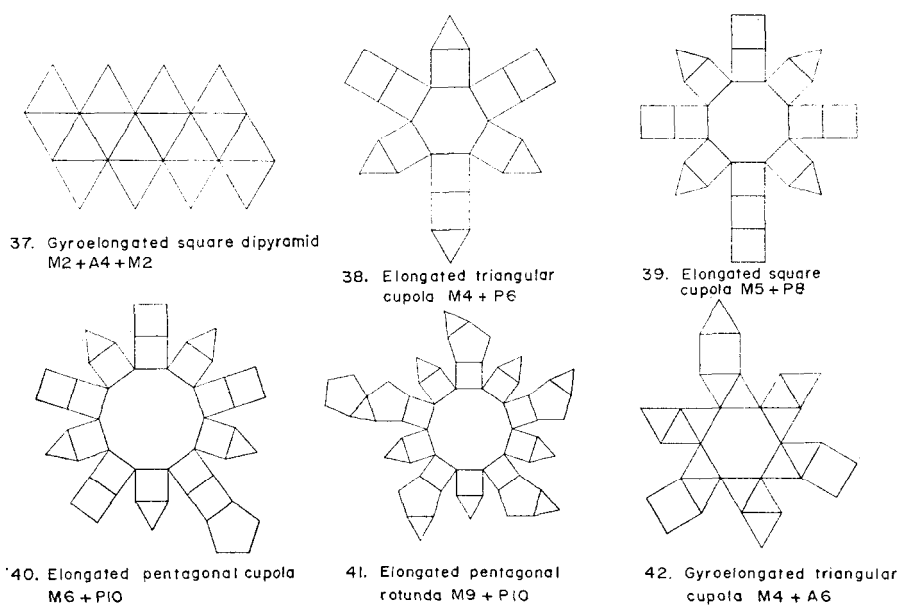


FIG. 14. Nets of regular-faced convex polyhedra 37-42.

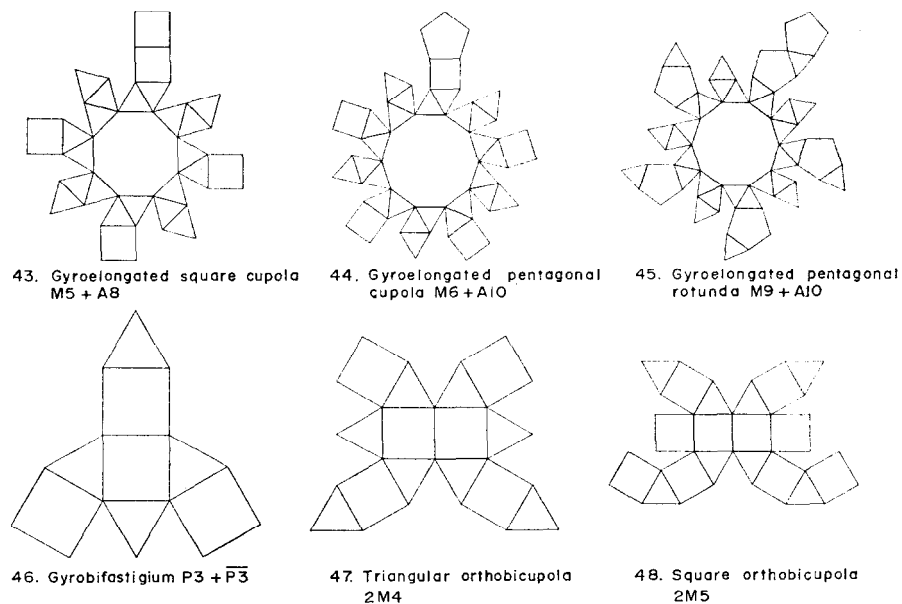


FIG. 15. Nets of regular-faced convex polyhedra 43-48.

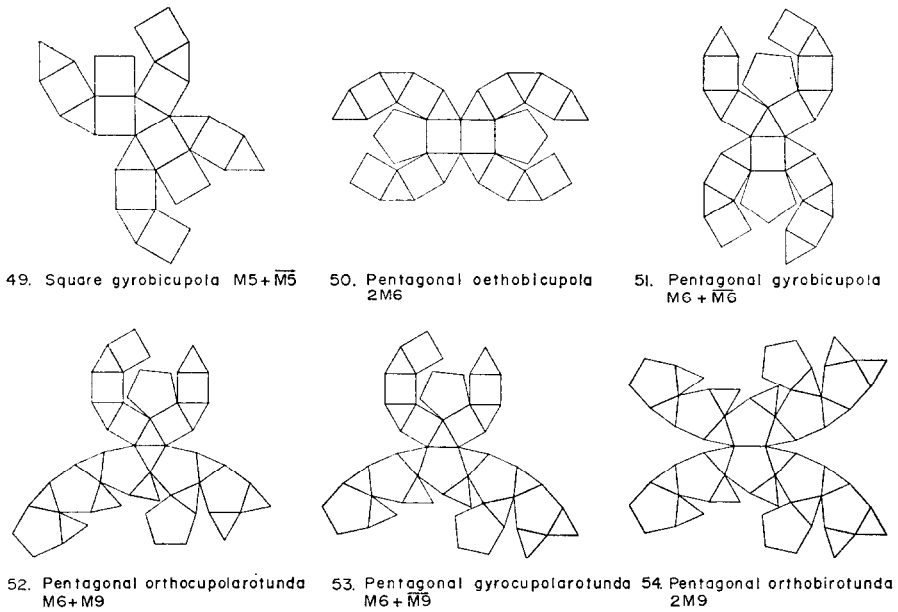


FIG. 16. Nets of regular-faced convex polyhedra 49-54.

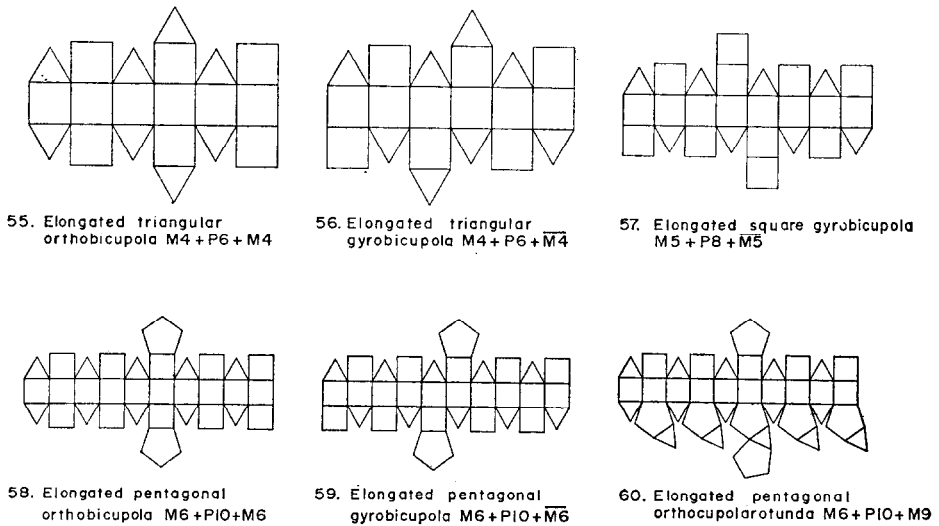


FIG. 17. Nets of regular-faced convex polyhedra 55-60.

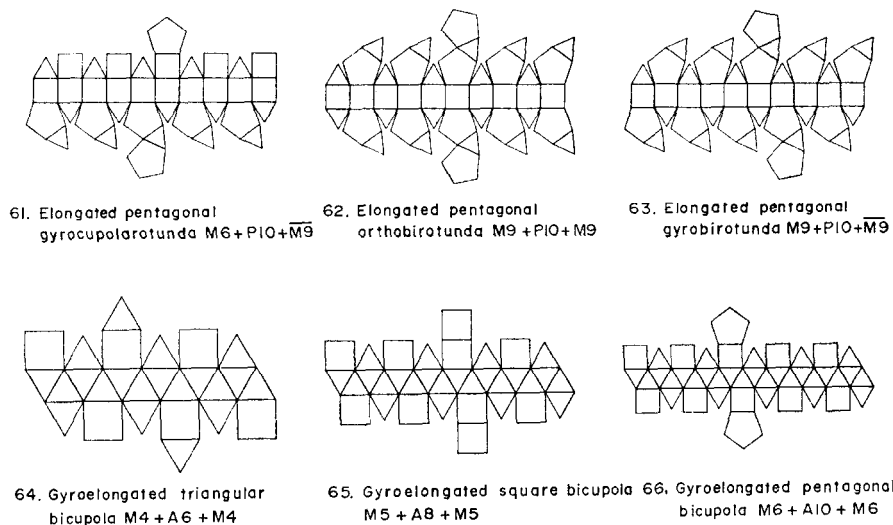


FIG. 18. Nets of regular-faced convex polyhedra 61-66.

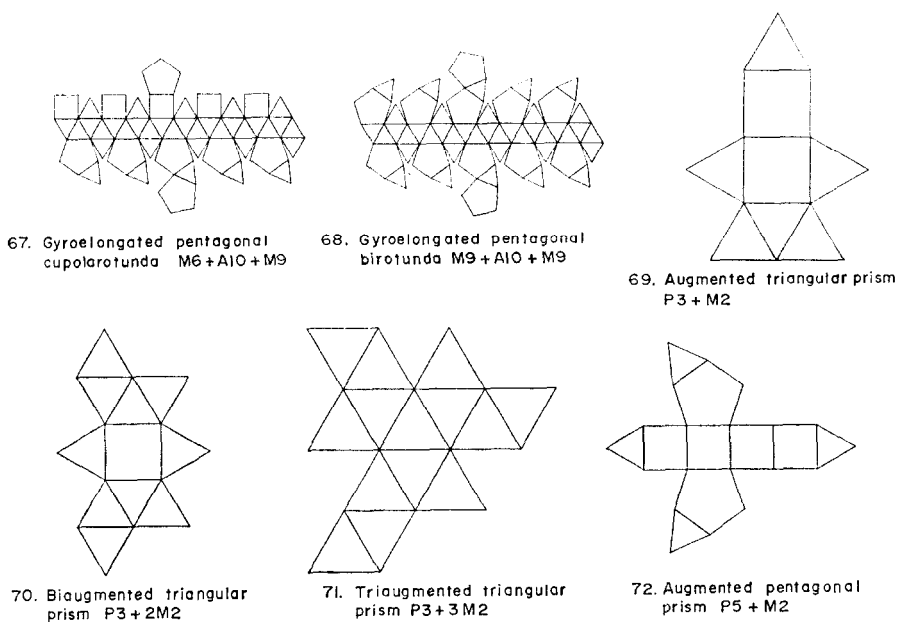


FIG. 19. Nets of regular-faced convex polyhedra 67-72.

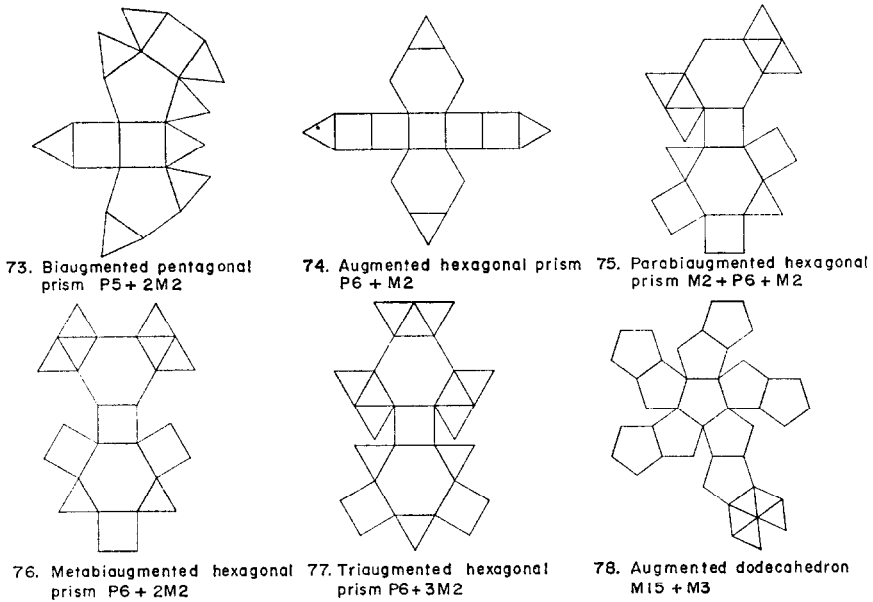


FIG. 20. Nets of regular-faced convex polyhedra 73-78.

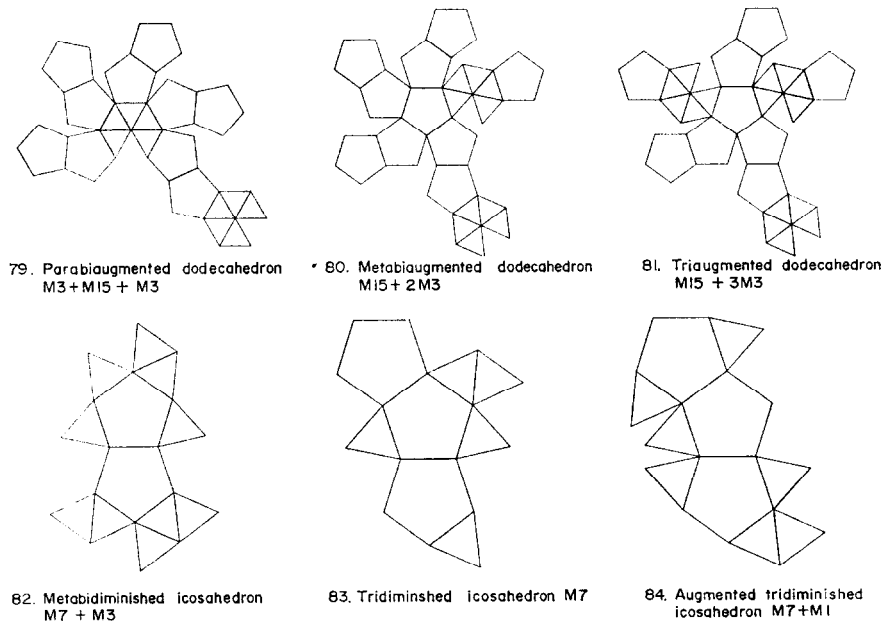


FIG. 21. Nets of regular-faced convex polyhedra 79-84.

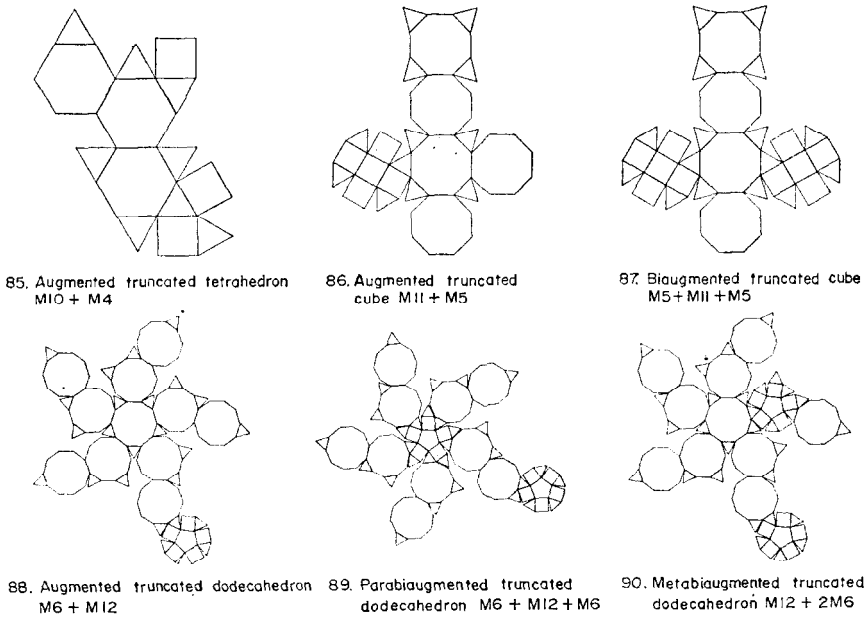


FIG. 22. Nets of regular-faced convex polyhedra 85-90.

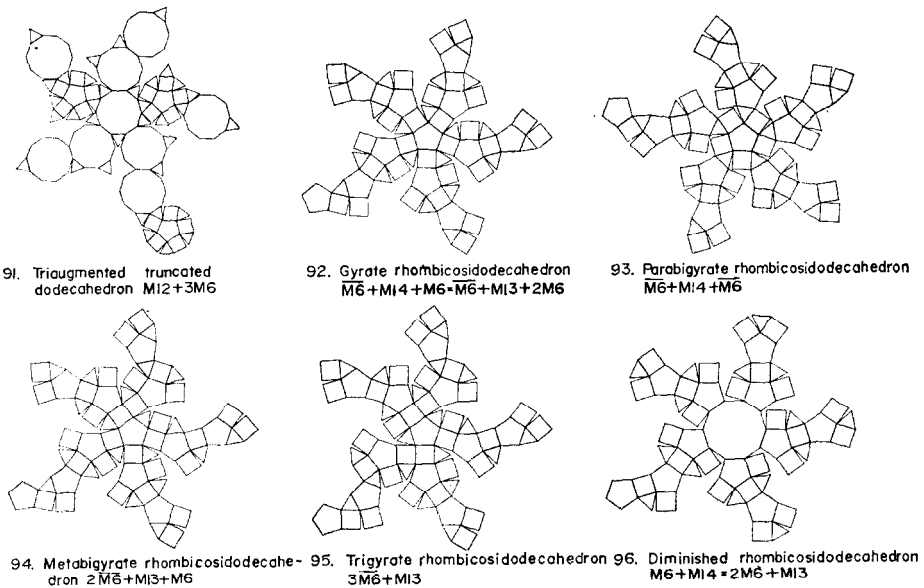


FIG. 23. Nets of regular-faced convex polyhedra 91-96.

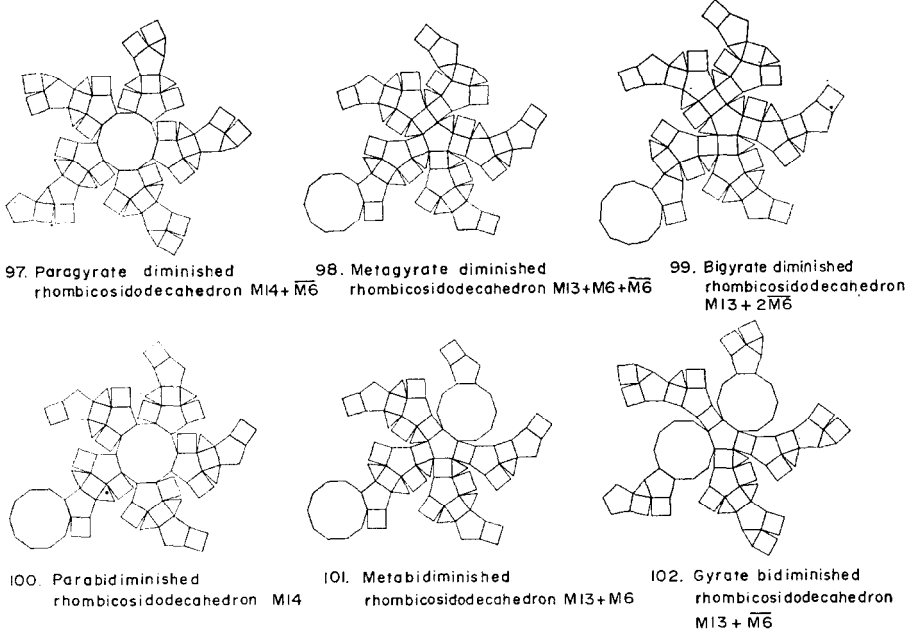


FIG. 24. Nets of regular-faced convex polyhedra 97–102.

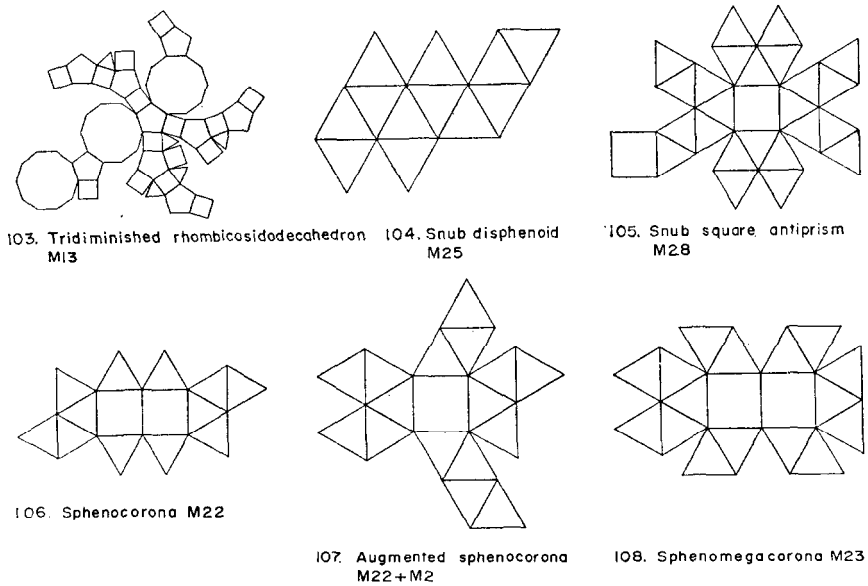


FIG. 25. Nets of regular-faced convex polyhedra 103–108.

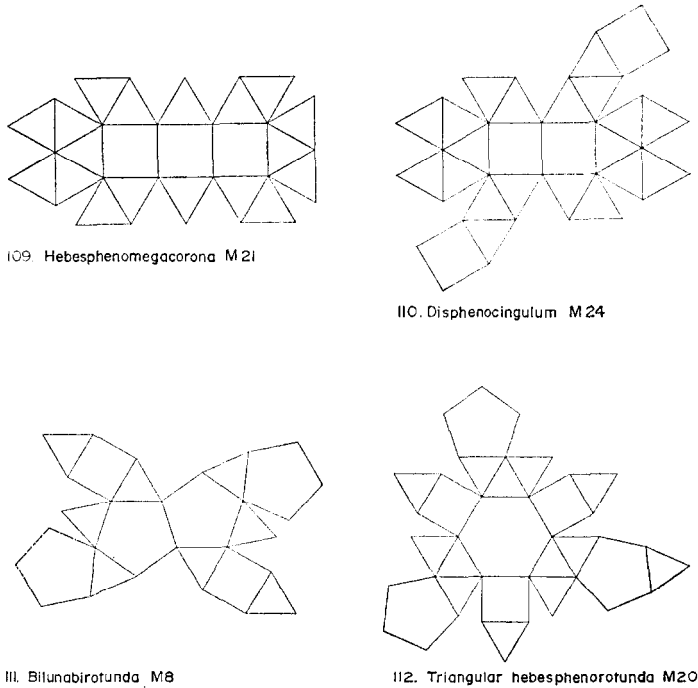


FIG. 26. Nets of regular-faced convex polyhedra 109–112.

(d) An n -gonal cupola has n triangles and n squares separating an n -gon and a $2n$ -gon; n can be 3, 4 or 5.

(e) A pentagonal rotunda has 10 triangles and 5 pentagons separating a pentagon and a decagon.

(f) A cupola and a rotunda put together form a cupolarotunda.

(g) Elongated means adjoined to an appropriate prism.

(h) Gyroelongated means adjoined to an appropriate antiprism.

(i) Two identical solids joined together are given the prefix “bi” or “di”.

(j) If two solids are joined together so that one of the two bases is the orthogonal projection of the other, as in a prism, then the prefix “ortho” is used.

(k) If two solids are joined together so that one of the two bases is turned relative to the other, as in an antiprism, then the prefix “gyro” is used.

(l) Two triangular prisms can be joined together to form a gyrobi-fastigium.

(m) Polyhedra can be augmented by adjoining solids to them.

(n) Polyhedra can be diminished by removing pieces of them.

(o) The prefix “tri” is used when three pieces are added or subtracted from a solid.

(p) Where there are two different ways of adjoining, removing or turning a pair of pieces, they are distinguished by prefixes “para” if opposite each other, and “meta” if not.

(q) A lune is a complex consisting of two triangles attached to opposite sides of a square.

(r) The prefix "spheno" refers to two adjacent lunes.

(s) The prefix "hebespheno" indicates a complex of two lunes separated by a third lune.

(t) The suffix "corona" refers to a crownlike complex of eight triangles.

(u) The suffix "megacorona" refers to a crownlike complex of twelve triangles.

(v) "Cingulum" indicates a belt of twelve triangles.

The names for the Platonic and Archimedean solids are ones that are frequently used.

Define the sphericity (designated as "Sphere" in Table IV) of a convex polyhedron to be the ratio of the volume of the polyhedron and the volume of a sphere with the same surface area. That is,

$$\text{Sphericity} = 6\pi^{1/3}V/A^{2/3} \quad (8)$$

and is a constant between zero and one. *Rfcp* vary from ones that are almost spherical to ones that are quite flat. To characterize this property we can use the sphericity, keeping in mind that for a given surface area a sphere has the maximum volume. Excluding prisms and antiprisms the flattest *rfcp* is the pentagonal cupola (No. 25) with a sphericity of 0.3085. As might be expected, the solid with the largest sphericity is the uniform polyhedron with the largest number of faces. It is the snub icosidodecahedron, which has 92 faces and a sphericity of 0.9731.

The solid having the fewest number of faces, edges, vertices and the least surface area and volume is the tetrahedron. The solid having the largest number of edges and vertices and the greatest surface area and volume is the truncated icosidodecahedron (No. 16).

Excluding the prisms and antiprisms, *rfcp* exist with from 4 to 92 faces, 4 to 120 vertices, 6 to 180 edges, surface areas of 1.732051 to 174.292030 and volumes of 0.117851 to 206.803399.

IV. Drawing of Nets and Construction of Models

Models of the *rfcp* were constructed and nets were then drawn from them. New models were then constructed from the nets to verify the accuracy of the nets. Nets of the Platonic solids (Nos. 1–5) abound in the literature. Cundy and Rollett's work contains nets of all the Archimedean solids (Nos. 6–18) (14). Nets are not unique and the ones in this article were chosen for ease in construction and to aid in visualizing changes taking place due to operations such as elongation, gyration, diminishing, etc. A configuration table was constructed for the net of each polyhedron. This was used as data for a computer program that calculated the abscissa and ordinate of each vertex of the net and normalized the results so that the net would fit into a square of edge length 25 cm. The results presented in Fig. 8–26 were obtained by plotting the nets on an incremental plotter with a resolution of 0.06 mm.

A method of construction of models of *rfcp* recommended by the author is:

- (a) Cut out the individual faces from a stiff material such as posterboard.
- (b) Lay out the faces according to the arrangement given in the appropriate net.
- (c) Apply tape to all the edges on what will become the inside of the polyhedron.
- (d) Join the appropriate edges to construct the polyhedron.
- (e) Glue all the edges from the outside along their entire length.

The only difficulty with this method is to apply tape from the inside of the polyhedron to the last face. This requires some practice! A simpler method, which does not produce as satisfactory a model, is to lay out the net on a single piece of posterboard and score and fold all the edges.

To build a complete set of models, one requires 1451 triangles, 840 squares, 375 pentagons, 75 hexagons, 25 octagons and 83 decagons for a total of 2848 faces. For an edge length of one, the surface area of the entire set of *rfcp* is 3109. To give some idea of the space requirements, a set was constructed with an edge length of 5 cm and it required approximately 23 m of shelf space for display. The smallest solid had a circumdiameter of 6.22 cm and the largest 36.62 cm.

References

- (1) G. C. Shepard, "Twenty problems on convex polyhedra", *Math. Gaz.*, Vol. 52, pp. 136-147, 1968.
- (2) H. S. M. Coxeter, M. S. Longuet-Higgins and J. C. P. Miller, "Uniform polyhedra", *Phil. Trans. Roy. Soc. London*, Ser. A, 254, pp. 401-450, 1954.
- (3) H. S. M. Coxeter, "Regular Polytopes", London, Methuen, 1947.
- (4) B. Grünbaum, "Convex Polytopes", New York, Interscience, pp. 412-416, 1967.
- (5) L. Fejes Tóth, "Regular Figures", New York, Pergamon Press, 1964.
- (6) V. A. Zalgaller, "Convex polyhedra with regular faces", *Seminars in Mathematics V. A. Steklov Math. Ins. Leningrad*, 2, English trans. Consultants Bureau, New York, 1969.
- (7) N. W. Johnson, "Convex polyhedra with regular faces", *Can. J. Math.*, Vol. 18, pp. 169-200, 1966.
- (8) V. A. Zalgaller, "Regular polyhedra", *Vestnik, Leningrad Univ. Ser. Mat. Mek. Astron.*, Vol. 18, pp. 5-8, 1963.
- (9) V. A. Zalgaller, "On regular-faced polyhedra", *Vestnik, Leningrad Univ. Ser. Mat. Mek. Astron.*, Vol. 20, pp. 150-152, 1965.
- (10) B. Grünbaum and N. W. Johnson, "The faces of a regular-faced polyhedron", *J. London. Math. Soc.*, Vol. 40, pp. 577-586, 1965.
- (11) D. W. Grace, "Computer search for non-isomorphic convex polyhedra", *Tech. Rep. C515*, Comp. Sci. Dept. Sch. of Humanities and Sciences, Stanford Univ., 1965.
- (12) B. Grünbaum, "Some analogues of Eberhard's theorem on convex polytopes", *Israel J. of Math.*, Vol. 6, pp. 398-411, 1968.
- (13) U. A. Griazioti, "Polyhedra, The Realm of Geometric Beauty", San Francisco, Holden-Day, 1962.
- (14) H. M. Cundy, and A. P. Rollett, "Mathematical Models", 2nd ed., London, Oxford University Press, pp. 101-115, 1962.